5.4 Trees

A **Cycle** is a path that begins and ends at the same vertex.
(where have we seen this before? A cycle is just a circuit)

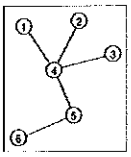
A **Tree** is a connected graph with no cycles.

Is the graph a tree?

1. 

2. 

3. 

4. 

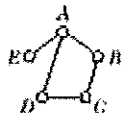
You have already had experience with trees... remember the brute force method for finding the shortest circuit for the traveling salesman? You drew trees!

5. Draw a tree to represent the following situation: tossing one penny and rolling one die.

6. Draw a tree to represent the following situation: A family has three children. How many outcomes of the gender of each child are possible?

5.4 Practice

Is the graph a tree? If the graph is not a tree explain why not.

1. 

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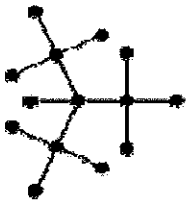
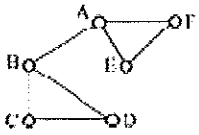
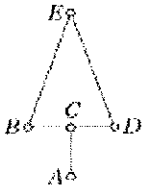
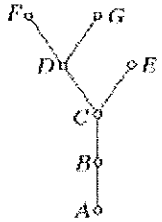
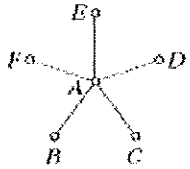
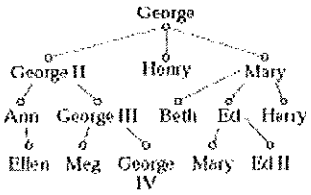
5. 

6. 

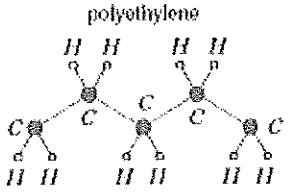
7. 

8. 

9. 

10. 

11. 

12. 

Draw a tree to represent the following situations.

13. You are at a carnival. One of the carnival games asks you to pick a door and then pick a curtain behind the door. There are 3 doors and 4 curtains behind each door. How many choices are possible for the player?

14. The 4 aces are removed from a deck of cards. A coin is tossed and one of the aces is chosen.

15. There are 3 trails leading to Camp A from your starting position. There are 3 trails from Camp A to Camp B. How many different routes are there from the starting position to Camp B?

16. A spinner has 4 equally likely regions numbered 1 to 4. The arrow is spun twice.

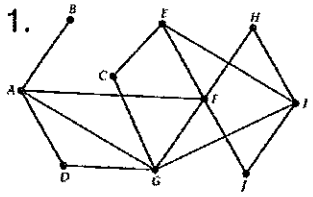
17. There are two identical bottles. One bottle contains 2 green balls and 1 red ball. The other contains 2 red balls. A bottle is selected at random and a single ball is drawn.

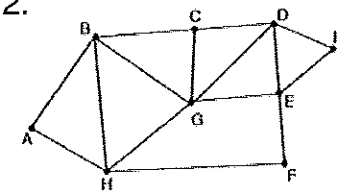
18. Anne was REALLY hungry last night and decided to order the Jumper Burger Dinner instead of just the Burger by itself! Burger dinners come with your choice of 1 drink and 2 side-dishes. Drinks that you can buy at the Marsburg Burger Bar are: Phobos Cola, Deimos Tea, or Water. Side-dish choices are: salad, french fries, or vegetable of the day.

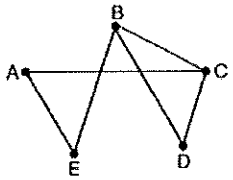
5.5 Minimum Spanning Trees (day 1)

A **spanning tree** of a connected graph is a subgraph that contains every vertex of the graph.

Find a spanning tree for each of the following graphs.

1. 

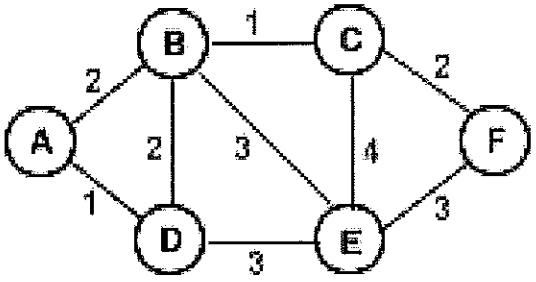
2. 

3. 

A **minimum spanning tree** is a spanning tree with the smallest possible weight.

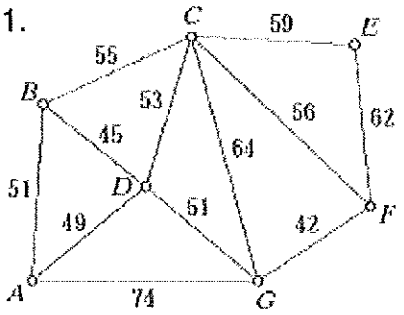
Kruskal's Method for finding a minimum spanning tree:

Step 1: list the edges from shortest to longest.
Step 2: darken the first edge on the list.
Step 3: look at the next edge on the list. If you can darken it and not create a cycle then darken it. If a cycle is created then do not darken the edge.
Step 4: continue down the list darkening edges until each vertex has been reached. Remember, a tree does not have cycles so be careful to avoid creating cycles.



Find a minimum spanning tree using Kruskal's methed.

1.



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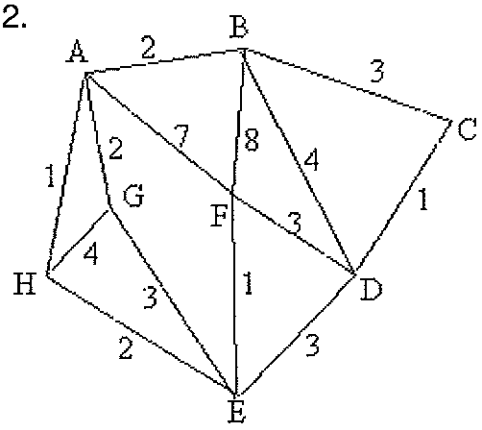
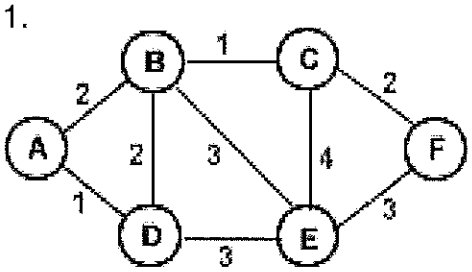


5.5 Minimum Spanning Trees (day 2)

Prim's Method for finding a minimum spanning tree:

Step 1: darken the shortest edge and circle both vertices (break ties however you choose)
Step 2: find the shortest edge between a circled vertex and an uncircled vertex, darken it, and circle the uncircled vertex
Step 3: repeat step 3 until all vertices are circled

1.



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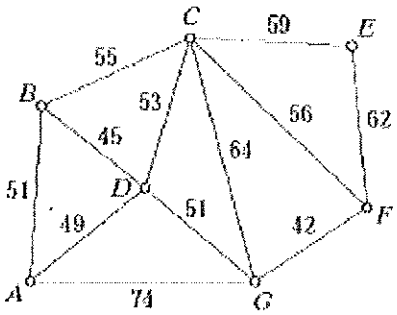

3.